

銘傳大學 99 學年度研究所碩士班招生考試

資訊工程學系碩士班與資訊傳播工程學系碩士班

第二節

資訊數學(含離散數學、線性代數)試題

(第 | 頁共 | 頁) (限用答案本作答)

可使用計算機 不可使用計算機

1. Prove that $\sqrt{2}$ is irrational. (10%)
2. Let G be the set of all nonzero real numbers and let $a*b = (ab)/2$. Show that $(G, *)$ is an Abelian group. (10%)
3. Let $A = \{x \mid x \text{ is an integer and } x^2 < 16\}$. Identify each of the following as true or false. (10%)
 - (a) $\{1, 3, 5\} \subseteq A$
 - (b) $\{-3, -2, -1\} \subseteq A$
 - (c) $\{\} \subseteq A$
 - (d) $\{x \mid x \text{ is an integer and } |x| < 5\} \subseteq A$
 - (e) $A \subseteq \{-3, -2, -1, 0, 1, 2, 3\}$
4. Let $S = (1 \times 2 \text{ matrices}, \square)$, where $[x, y] \square [w, z] = [x+w, (y+z)/2]$. Determine which of the following properties hold for this structure. (10%)
 - (a) Closure
 - (b) Associative
5. Find an explicit formula for the sequence defined by $c_n = 3c_{n-1} - 2c_{n-2}$ with initial conditions $c_1 = 5$ and $c_2 = 3$. (10%)
6. Consider the following homogeneous system of linear equations and its general solution. The solutions will form a subspace W of \mathbf{R}^4 . Find a basis for W and give its dimension.

$$\begin{aligned}x_1 + x_2 - 2x_3 + 4x_4 &= 0 \\2x_1 + x_2 - 4x_3 + 5x_4 &= 0 \\3x_1 + 2x_2 - 6x_3 + 9x_4 &= 0\end{aligned}$$

General solution is $(2r - s, -3s, r, s)$. (10%)

7. Find the matrix that maps $\mathbf{R}^2 \rightarrow \mathbf{R}^2$ such that $\begin{bmatrix} 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 3 \\ -2 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ 11 \end{bmatrix}$. (10%)
8. Let A be an invertible matrix. Show that the eigenvalues of A^{-1} are the inverses of the eigenvalues of A . Prove that A and A^{-1} have the same eigenvectors. (10%)
9. Find bases for the kernel and range of the transformation defined by the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 3 & 1 \end{bmatrix}$ (10%)
10. Find the coordinate vectors of $\mathbf{u} = (4, 5)$ relative to the following basis B of \mathbf{R}^2 :
 $B = \{(2, 1), (-1, 1)\}$. (10%)

試題完