

銘傳大學 96 學年度研究所碩士班招生考試
應用統計資訊學系碩士班
第一節

線性代數試題

(第 1 頁共 1 頁)
(限用答案本作答)

1. (20%) Let $A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{bmatrix}$.

- (a) Find the eigenvalues and associated eigenvectors of A .
 - (b) Find an orthogonal matrix P such that $P^{-1}AP = D$, where D is a diagonal matrix.
 - (c) Find A^8 .
2. (15%) Let W be the subset of \mathbb{R}^4 which is spanned by $u_1 = (1, -2, 0, 1)$, $u_2 = (-1, 0, 0, -1)$, $u_3 = (1, 1, 0, 0)$.

- (a) Use the Gram-Schmidt process to find an orthonormal basis for W .
- (b) Find the orthogonal projection of the vector $(2, 1, 0, 1)$ on W .

3. (15%) Let $S = \{v_1, v_2, v_3, v_4\}$, where $v_1 = (1, -2, 0, 3, -4)$, $v_2 = (3, 2, 8, 1, 4)$, $v_3 = (2, 3, 7, 2, 3)$, $v_4 = (-1, 2, 0, 4, -3)$, and let V be the subspace of \mathbb{R}^5 given by $V = \text{span } S$. Find a basis for V and V^\perp .

4. (10%) Consider the transition matrix $A = \begin{bmatrix} 0 & 0.2 & 0 \\ 0 & 0.3 & 0.3 \\ 1 & 0.5 & 0.7 \end{bmatrix}$. Show that A is regular and find its steady-state vector.

5. (10%) $A_{n \times n}$ is a symmetric matrix. Show that eigenvectors that are associated with distinct eigenvalues of A are orthogonal.

6. (15%) Let $L: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined by $L = \left(\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \right) = \begin{bmatrix} a_1 + a_3 \\ a_1 + a_2 + 2a_3 \\ 2a_1 + a_2 + 3a_3 \end{bmatrix}$.

- (a) Is L onto?
- (b) Find a basis for range L .
- (c) Find $\ker L$

7. (5%) Let $A = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & 1 & 2 & 1 \\ 2 & -2 & 5 & 2 \\ 3 & 3 & 0 & 0 \end{bmatrix}$. Compute $\det(A^{-1})$.

8. (10%) Let A be a $k \times k$ symmetric and idempotent matrix of rank r . Show that the rank of A is equal to its trace.

試題完