

離散數學 試題

(限用答案本作答)

每題 20 分

1. Given three arbitrary sets $A, B, C \subseteq U$. Prove or disprove the following statements.
 - (a) $A \oplus A = \phi$
 - (b) $(A - B) - C = A - (B \cup C)$
 - (c) $(A \cup B) \cap (B \cup C) \cap (A \cup C) = A \cap B \cap C$
 - (d) $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$
2. Let $(S, *)$ be the group of all real numbers except -1 under the operation $*$ defined by $a * b = a + b + ab$. Show that $(S, *)$ is isomorphic to the group \mathbb{R}^* of nonzero real numbers under multiplication.
3. Let R be a binary relation. Let $S = \{(a, b) \mid (a, c) \in R \text{ and } (c, b) \in R \text{ for some } c\}$. Show that if R is an equivalence relation, then S is also an equivalence relation.
4. Solve the following recurrence relations.
 - (a) $a_{n+1} - a_n = 3^n, n \geq 0, a_0 = 1$.
 - (b) $a_{n+2} - 3a_{n+1} + 2a_n = 0, n \geq 0, a_0 = 1, a_1 = 6$.
5. (a) If a connected planar simple graph has e edges and v vertices with $v \geq 3$ and no circuits of length 3. Show that $e \leq 2v - 4$.
 (b) How many regions would there be in a planar graph with 10 vertices each of degree 3?

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