銘傳大學九十二學年度資訊傳播工程學系碩士班招生考試

第四節

機率 試題

1. Suppose that the random variables X, Y, and Z have the following joint probability density function .

$$f(x,y,z) = \begin{cases} 1, 0<x<y<1, 0<z<2, \\ 0, \text{otherwise.} \end{cases}$$

Compute P(4X>Y │3Z<1). (15%)

2. The random variable X has the following probability density function.

$$f(x) = \begin{cases} \frac{1}{2}x^2 e^{-x}, x > 0 \\ 0, \quad x \le 0 \end{cases}$$

Compute E[$X^{\frac{1}{2}}$]. (15%)

3. Let X have a Poisson distribution, with parameter $\theta > 0$ .Using Chebychev's inequality to prove that

$$P(X \le \frac{\theta}{2}) \le \frac{4}{\theta}. \qquad (15\%)$$

4. Let the random variable X have the density function $fx(x) = \lambda e^{-\lambda x}, x \ge 0, \lambda > 0$. Show that X has the lack of memory property

$$P(X > t1 + t2 | X > t1) = P(X > t2), t1, t2 > 0 \quad (10\%)$$

5. Let X1, X2 be a random sample from the standard normal distribution, N(0,1). Find the distribution of the following random variables.

   (a) $\frac{X1 - X2}{\sqrt{2}}$ (3%)

   (b) $(X1 + X2)^2 \ / \ (X1 - X2)^2$ (4%)

   (c) $(X1 + X2)/\sqrt{(X1 - X2)^2}$ (4%)

   (d) $X1^2 \ / \ X2^2$ (4%)

6. Let X be a random variable with the following probability distribution

$$f(x) \begin{cases} (\alpha+1)x^{\alpha}, 0 < x < 1, \alpha > -1 \\ 0, \quad \text{otherwise.} \end{cases}$$

Find the maximum likelihood estimator of α, based on a random sample of size n. (15%)

7. Let $X1, X2, \ldots Xn$ be a random sample of a continuous random variable with cumulative distribution function F(x). Find

$E[F(x_{(n)})] when X_{(n)} = \max\{X1, X2 \ldots Xn\}$. (15%)