銘傳大學九十一學年度資訊管理研究所碩士班招生考試
資訊傳播工程

第四節

離散數學 試題

1. (20pts.) For the following statements the universe comprises all nonzero integers. Determine the truth or falsity of each statement. If a statement is false, give a counterexample.
   (a) $\exists x \forall y [xy = 1]$
   (b) $\forall x \forall y [a > b \to a^2 > b^2]$
   (c) $\exists x \exists y [(3x - y = 7) \wedge (2x + 4y = 3)]$
   Let the universe for the variables in the following statements consist of all real numbers. Please negate and simplify these statements.
   (d) $\forall x \forall y [(x < y) \to \exists z (x < z < y)]$
   (e) $\lim_{x \to a} f(x) = L \Leftrightarrow \forall \varepsilon > 0 \exists \delta > 0 \forall x [(0 < |x - a| < \delta) \to (|f(x) - L| < \varepsilon)]$
2. (15pts.)(a) Let R_1 be a partial order relation on A and R_2 be a partial order relation on B. On A x B, we define relation R by (a, b) R(x, y) if a R_1 x and b R_2 y. Show that R is a partial order.
   （b） If A = A1∪A2∪A3, where $A_1 = \{1, 2\}$, $A_2 = \{3, 4\}$ and $A_3 = \{5\}$, define relation R on A by x Ry if x and y are in the same subset Ai, 1≦i≦3. Explain whether R is an equivalence relation.
3. (5pts.)Show that any subset of size six from the set S = {1, 2, 3,…, 9} must contain two elements whose sum is 10.
4. (10pts.) Let f: Z□N be defined by
   $$f(x) = \begin{cases} 2x - 1, & if \quad x > 0 \\ -2x, & for \quad x \le 0 \end{cases}$$
   (a) Prove that f is one-to-one and onto.
   (b) Determine f^{-1}.
5. (10pts.)(a) The sequence of the Lucas numbers is defined recursively by
   1) $L_0 = 2$, $L_1 = 1$; and
   2) $L_n = L_{n-1} + L_{n-2}$, for $n \in Z^+$ with n≧2
   Prove that for $n \geq N$
   $$\sum_{i=0}^{n} Li = L_{n+2} - 1$$
   (b) Let T_n denote the number of movements of discs in the Hanoi

Tower problem with n discs. Define the recurrence relation for the recursive algorithm that you may design. Solve the recurrence relation for T_n.

6. (20pts.)(a) Give the definition of a tree. (Suppose that G=(V, E) is a undirected graph.)

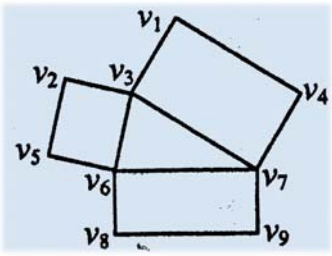
    (b) Prove that in any tree T= (V, E), |V| = |E| + 1.

    (c) Let T = (V, E) be a tree with |V| = n. How many distinct paths are there in T?

    (d) Describe an algorithm you used to find a minimum spanning tree. What is the time complexity of your algorithm? Explain.

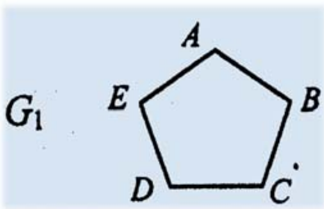
7. (10pts.)(a) What is a bipartite graph?

    (b) Prove that the following graph is not bipartite.



8. (10pts.)(a) What is “graph isomorphism”?

    (b) Draw a graph G_2 which is isomorphic to the following graph G_1.



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