銘傳大學九十一學年度資訊傳播工程研究所碩士班招生考試

第四節

線性代數 試題

一、 Find the values of a such that the following linear system has (1)exactly one solution, (2)infinitely many solution, and (3)no solution. 15%

$$\begin{aligned} x + \quad\quad\quad & z = 4 \\ 2x + \quad y + \quad\quad & 3z = 5 \\ -3x - 3y + (a2 - 5a)z & = a - 8 \end{aligned}$$

二、 Find a 3x3 matrix A having eigenvalues 2, 4, 1 and corresponding elgenvectors: 10%

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

三、 Let S = {(l,2),(0,l)} and T = {(1,1),(2,3)} be bases for R_2 .Let x == (l,5) and y == (5,4). 15%

(1) Find the coordinate vectors of x and y with respect to the basis T.

(2) What is the transition matrix $P_{S<-T}$, from the T - to the S - basis?

(3) Find the transition matrix $Q_{T<-S}$ from the S - to the T - basis?

四、 Let P3 be the vector space of real polynomials with degrees≦3. Define the inner product <p, q> = $\int_{-1}^{1} p(x)q(x)dx$.

(1) Find an orthonormal basis of P_3. 15%

(2) Write $r(x) = 2x^2 + 3x - 5$ as a linear combination of the basis obtained in part (l). 5%

五、 Let L:M_{nn} -> M_{nn} be defined by $L(A) = A + A^T$, where A^T is the transpose of A and M_{nn} is the set of all n x n matrices.

(1) Show that L is a linear operator from M_{nn} into M_{nn}. 10%

(2) Find the range space of L and the kernel space of L. 10%

六、 Let U and W be subspaces of a vector space V. State whether the following is also a subspace of V. Prove or give a counter example.

(1) $U \cap W$ 10%

(2) $U \cup W$ 5%

(3) $(V - U) \cap W$ 5%

試題完