

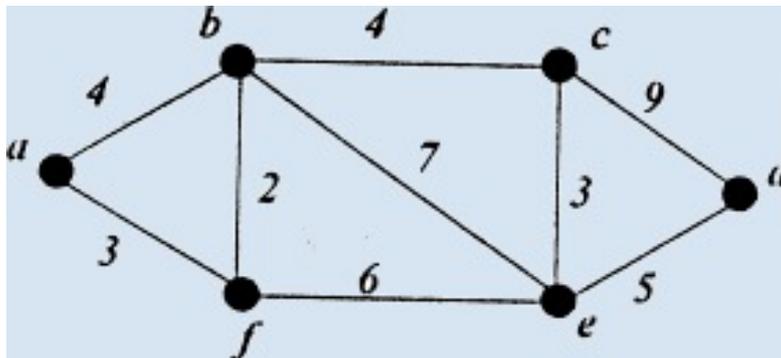
銘傳大學八十七學年度資訊管理研究所碩士班招生考試

第二節

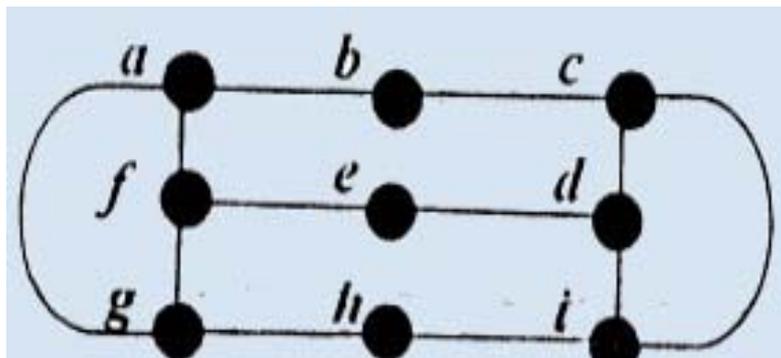
離散數學 試題

The points are indicated at the rear of the (sub)problem.

1. (a) The edges of complete graph K_6 are colored in either red or blue. Prove that there must be a red triangle or a blue triangle. {10%} (b) Does the above property hold for complete graph K_5 ? If so, prove it; otherwise, disprove it by giving a counter example. {5%}
2. For the graph given below, find (a) a minimum spanning tree, (b) an Eulerian path, (c) a maximal independent set, and (d) a minimal dominating set. {3% each}



3. Is there a Hamiltonian path in the graph given below? Why? {6%}



4. Let Θ denote a binary relation defined on set $A = \{2, 3, 4, 6, 12, 16, 18, 24, 32\}$ by divisibility. i.e. for $x, y \in A$, $x \Theta y$ if and only if x divides y . (a) Which of the four properties “reflexive, transitive, symmetric, and anti-symmetric” are satisfied? {5%} (b) Draw the corresponding Hasse Diagram. {5%}
5. Define recurrence relations, including their initial conditions, for the following problems: (You do not have to solve it.) (a) Let $H(n)$ denote the number of moves when dealing with n discs in Hanoi Tower problem. {5%}

(b) A code word defined on the alphabet $\{0, 1, 2, 3\}$ is legitimate if and only if it has an even number of 0's. Let $C(n)$ denote the number of legitimate code words of length n . {5%}

6. Suppose that G is a permutation group defined on set $A = \{1, 2, 3\}$. Let S be a binary relation defined on set A such that for $x, y \in A$, xSy if and only if there is a permutation π in G such that $\pi(x)=y$. Show that S is an equivalence relation. {10%}

7. Find the number of non-negative integer solutions to the following equations

(a) $x_1 + x_2 + x_3 + x_4 = 40, x_1 \geq 7$; {4%}

(b) $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 50, x_1 + x_2 + x_3 = 12$. {6%}

8. Show, by a combinatorial proof, that {8%}

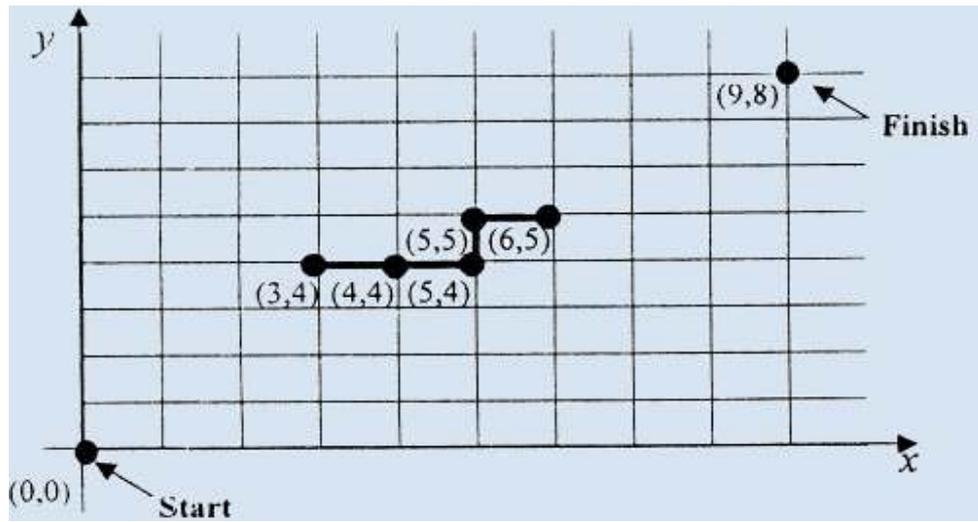
$$\binom{m+n}{r} = \binom{n}{0} \binom{m}{r} + \binom{n}{1} \binom{m}{r-1} + \binom{n}{2} \binom{m}{r-2} + \dots + \binom{n}{r} \binom{m}{0}$$

9. There are six committee meeting to be scheduled with three available meeting times. The following table summarizes which committees have a member in common. The problem seeks to schedule all meetings such that committees have a common member do not use a common time. How do we treat this issue as a graph-coloring problem? {9%}

	Finance	Environment	Health	Transportation	Education	Housing
Finance	0	0	0	0	0	1
Environment	0	0	1	0	1	0
Health	0	1	0	1	1	1
Transportation	0	0	1	0	0	1
Education	0	1	1	0	0	1
Housing	1	0	1	1	1	0

The entry (i, j) is 1 if committee i and j have a common member, and 0 otherwise.

10. On the x - y plane, only two moves $(x, y) \rightarrow (x+1, y)$ and $(x, y) \rightarrow (x, y+1)$ are allowed. (a) How many ways can we move from $(0, 0)$ to $(9, 8)$? {4%} (b) How many ways can we move from $(0, 0)$ to $(9, 8)$ without going through the darkened path indicated below? {6%}



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